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The Methods of J. S. Mill

John Stuart Mill (1806-73) was the most important philosopher writing in English in the nineteenth century. Close Reasoners will already be familiar with his 1859 essay, *On Liberty,* and its stout defence of freedom of thought and expression. Present day discussions of such issues frequently include such stirring quotes as:

- John Stuart Mill (*On Liberty,* 1859)

> The sole end for which mankind are warranted, individually or collectively, in interfering with the liberty of action of any of their number is self-protection.

and:

> If all mankind minus one were of one opinion, and only one person were of the contrary opinion, mankind would be no more justified in silencing that person than he, if he had the power, would be justified in silencing mankind.

He made important contributions in other branches of philosophy as well as in the practical world of political reform.

Our concern here, however, is with his *A System of Logic, Ratiocinative and Inductive, being a Connected View of the Principles of Evidence and the Methods of Scientific Investigation,* generally known simply as Mill's *Logic,* published in 1843. This fat book (The 7th edition is in two volumes of about 550 pages each) covered the whole field of logic (*ratiocinative = deductive*), but our interest in this ***note*** is only in the part on induction. Mill was what philosophers call an *empiricist,* believing that knowledge rests on experience and, in the tradition of Francis Bacon and John Locke, he saw that fact as the key to the advancement of science, and wanted it incorporated into Logic. The degree to which he succeeded in doing this has been much debated.

The book was very influential in its time. A biographer says:

- Alan Ryan (1974)

> The *Logic* occupies a central place in Mill's writings, though it is not in the nature of the case a widely-read book today. It did exercise a considerable influence in its heyday: it went into eight editions, including a cheap edition for working-class readers, and it became a textbook for the study of logic in most English and many foreign universities. (Alan Ryan, *J.S.Mill* (London: 1974), p. 60)

And Mill himself was puzzled by the popularity of this book. He remarks in his *Autobiography* (1873):

- John Stuart Mill (*Autobiography,* 1873)

> How the work came to have, for a work of this kind, so much success, and what sort of persons compose the bulk of those who have bought, I will not venture to say read, it, I have never thoroughly understood. But taken in conjunction with the many proofs which have since been given of a revival of speculation, speculation too of a free kind, in many quarters, and above all (where at one time I should have least expected it) in the Universities, the fact becomes partially intelligible.

Among those 'foreign universities' referred to by Ryan, and, if Mill is right, promoting free speculation, was the University of Toronto. Our library at The University of Western Ontario has a copy of a New York (1867) edition of the *Logic* which appears to have been used at Toronto as a textbook. It was owned by one A. Mackenzie in 1868, and, passing no doubt from student to student, again by one

Chs. Clarkson in 1875. Along the way helpful persons pasted into the book questions from the exams in Metaphysics and Ethics for 1873 and 1874, and added pages numbers where the answers might be found. Clarkson must have been in good shape when his time came.

John Venn, among many other later writers on logic, was an admirer of Mill. Venn lectured on inductive logic at Cambridge University, and he published a version of his lectures in 1879 under the title, *The Principles of Empirical or Inductive Logic* in 1879. In his *Preface* he remarks:

- John Venn (1st edition, 1879)

> As many readers will probably perceive, the main original guiding influence with me,—as with most of those of the middle generation, [Venn was 55 at this time] . . . —was that of Mill. . . . I still continue to regard the general attitude towards phenomena, which Mill took up as a logician, to be the soundest and most useful for scientific study.

But the popularity of Mill gradually faded; 18 years later in 1907, Venn, then 73 and retired, got out a second edition, he said in the new *Preface:*

- John Venn (2nd edition, 1907)

> As the reader will see, in the arrangement originally adopted it was throughout presupposed that Mill's Logic was familiar to my hearers, and that they had the work in their hands. Mill, in fact, dominated the thought and study of intelligent students to an extent which many will find hard to realize at the present day. And they mostly absorbed his teaching at first hand. . . . I am assured by those who are now more in contact with the modern student than I am that the former familiarity with the doctrines of Mill may still be taken for granted to a considerable extent, though doubtless it is now obtained, more than was formerly the case, through derivative channels.

The 'derivative channels' are still with us in any number of elementary logic textbooks, (and in this ***Note***), especially the treatment of "Mill's Methods", to which we will shortly turn.

Mill, rightly, attaches great importance to his work on inductive logic, and holds that it marks a significant advance over the views of his predecessors, chief among whom was Whately. In his *Preface* he says:

- John Stuart Mill (*System of Logic*, 1843)

> On the subject of Induction, the task to be performed [in this book] was that of generalizing the modes of investigating truth and estimating evidence, by which so many important and recondite laws have been aggregated to the stock of human knowledge. That this is not a task free from difficulty may be presumed from the fact, that even at a very recent period, eminent writers (among whom it is sufficient to name Archbishop Whately, . . .) have not scrupled to pronounce it impossible.

And to this Mill (in later editions) attaches a brave footnote in which he takes up the challenge:

> In the later editions of Archbishop Whately's *Logic*, he states his meaning to be, not that "rules" for the ascertainment of truths by inductive investigation cannot be laid down, or that they may not be "of eminent service," but that they "must always be comparatively vague and general, and incapable of being built up into a regular demonstrative theory like that of the Syllogism." (Book IV. ch. iv. s. 3.) And he observes, that to devise a system for this purpose, capable of being "brought into a scientific form," would be an achievement which "he must be more sanguine than scientific who expects." (Book IV. ch. ii. s. 4.) To effect this, however, being the express object of the present work which treats of Induction, the words in the text [quoted above] are no overstatement of the difference of opinion between Archbishop Whately and me on the subject.

So the battle is joined! Can Mill bring inductive reasoning into a demonstrative system comparable to the theory of the Syllogism? Is such a thing possible at all? Will Whately have the last laugh? Present day writers on these topics are less sanguine than Mill about reducing induction to a few simple rules, but many would hold that the situation is not nearly as difficult as Whately makes out. And all would agree that Mill's Methods are at least of 'eminent service'. More on this after we have looked at the Methods.

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The part of Mill's *Logic* relevant here is Book III, *Of Induction*, and especially Chapter VIII, *Of the Four Methods of Experimental Inquiry*. These are now known as *Mill's Methods*: **Agreement**, **Difference**, **Residues** and **Concomitant Variations.** Mill adds a fifth, a sort of combination of Agreement and Difference, which he calls the **Joint Method**, and summarizes them all in five 'regulative principles', which he calls *Canons*

Mill supposes that experiments can be represented as combinations of circumstances which may be divided antecedents and consequents. He uses capital letters for the former and small letters for the latter, and so uses "A B C *a b c*" to describe an experiment in which we put together A B and C, and get the results *a b c*.

MILL'S FIVE CANONS

First Canon
[Method of Agreement]

If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon.

Second Canon
[Method of Difference]

If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, that one occuring only in the former, the circumstance in which alone the two instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.

Third Canon
[Joint Method of Agreement and Difference]

If two or more instance in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.

Fourth Canon
[Method of Residues]

Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

Fifth Canon
[Method of Concomitant Variations]

Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

Suppose we do this for our first experiment, and then try a second in which A D E gives us *a d e*. This may be summed up as follows:

1. A B C *a b c*
2. A D E *a d e*

We may then reason, he says, that *b* and *c* are not results of A since we didn't get them from A in the second experiment. Likewise, *d* and *e* are not results of A since we didn't get them in the first experiment. So the effect of A has to be *a*. This is his **Method of Agreement**; A is the antecedent circumstance in which all the cases of *a* agree. He gives an illustration, drawn from chemistry:

> For example, let the antecedent A be the contact of an alkaline substance and an oil. This combination being tried under several varieties of circumstances, resembling each other in nothing else, the results agree in the production of a greasy and detersive or saponaceous [he is trying to say 'soap like'] substance: it is therefore concluded that the combination of an oil and an alkali causes the production of soap. It is thus we inquire, by the Method of Agreement, into the effect of a given cause.

We find, that is to say, that when we put oil and alkali together we always get soap, no matter what the other circumstances. We conclude that oil plus alkali is a sufficient condition for soap.

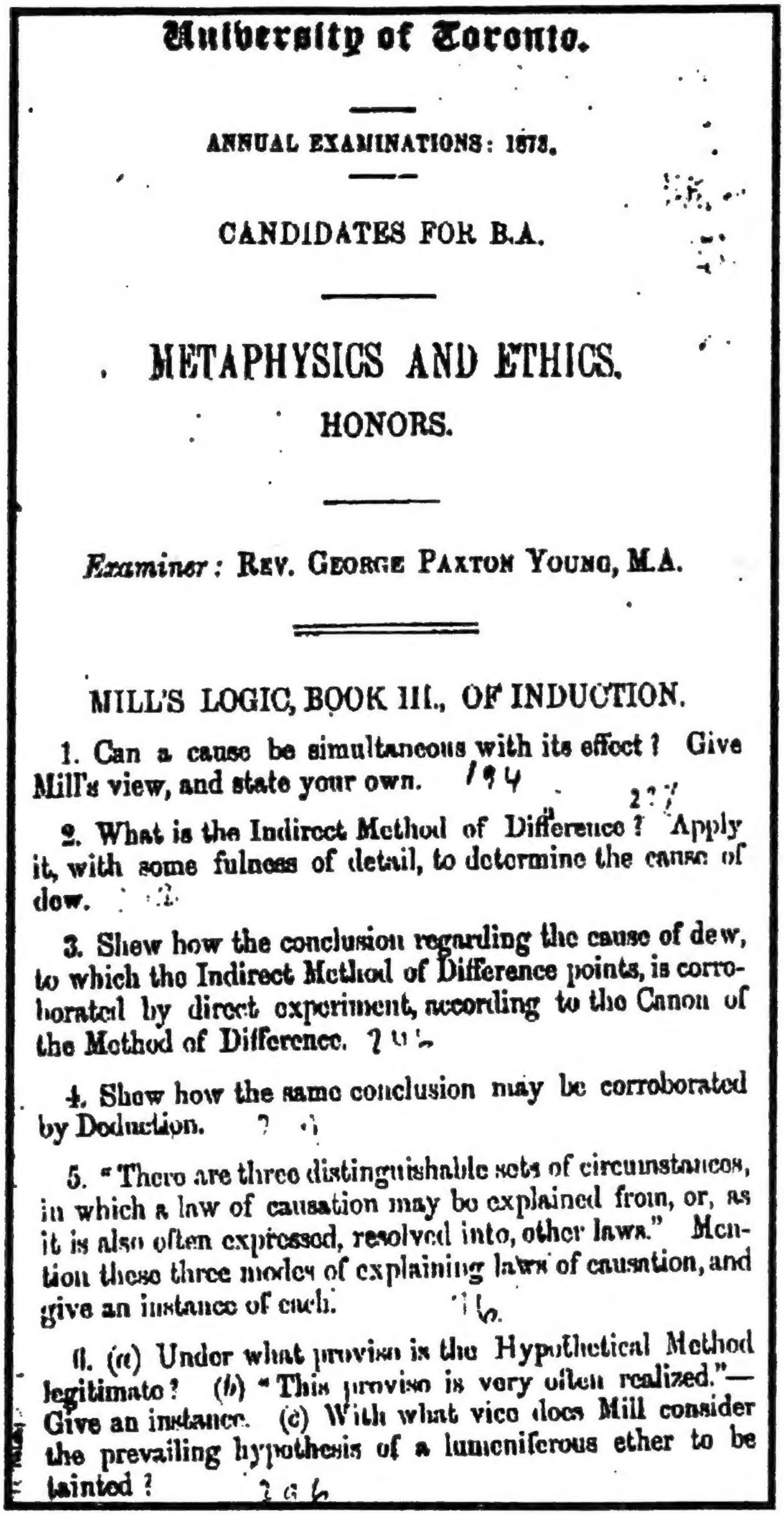
Here we were starting with the cause and asking what is the effect; we might (Mill doesn't) call it the *forward* use of the Method of Agreement. But the method works in reverse too, a *backward* use, for we can start with the effect and ask about the cause:

> For example, let the effect *a* be crystallization. We compare instances in which bodies are known to assume crystalline structure, but which have no other point of agreement; and we find them to have one, and as far as we can observe, only one antecedent in common: the deposition of a solid matter from a liquid state, either a state of fusion or of solution. We conclude, therefore, that the solidification of a substance from a liquid state is an invariable antecedent of its crystallization.

Here we find that, no matter what the circumstances, crystals come from a liquid to solid transformation, never from anything else, and so we conclude that the existence of a crystal is a sufficient condition for there having been such a transformation.

The difference between these two uses seems to be that in the forward use we try an antecedent circumstance in all sorts of combinations with other things and look to see if there is a common consequent, while in the backward use we study a consequent circumstance in different combinations and look to see if there is a common antecedent.

But, are their other ways of making soap? Can you solidify a liquid without getting a crystal? Here we turn to the **Method of Difference**, by looking at cases in which circumstances differ by the presence or absence of just one



University of Toronto.

ANNUAL EXAMINATIONS: 1878.

CANDIDATES FOR B.A.

METAPHYSICS AND ETHICS.

HONORS.

Examiner: Rev. George Paxton Young, M.A.

MILL'S LOGIC, BOOK III., OF INDUCTION.

1. Can a cause be simultaneous with its effect? Give Mill's view, and state your own. 194 227

2. What is the Indirect Method of Difference? Apply it, with some fulness of detail, to determine the cause of dew. [illegible]

3. Shew how the conclusion regarding the cause of dew, to which the Indirect Method of Difference points, is corroborated by direct experiment, according to the Canon of the Method of Difference. [illegible]

4. Show how the same conclusion may be corroborated by Deduction. [illegible]

5. "There are three distinguishable sets of circumstances, in which a law of causation may be explained from, or, as it is also often expressed, resolved into, other laws." Mention these three modes of explaining laws of causation, and give an instance of each. [illegible]

6. (*a*) Under what proviso is the Hypothetical Method legitimate? (*b*) "This proviso is very often realized."—Give an instance. (*c*) With what vice does Mill consider the prevailing hypothesis of a luminiferous ether to be tainted? 266

of the features of interest. You take away A, for example, and find that you lose a. The pattern is:

1. A B C $a\ b\ c$
2. B C $b\ c$

You conclude that you need to have A to get a, which is to say that A is a necessary condition for a. Mill thinks that this mode of reasoning is very common.

> It is scarcely necessary to give examples of a logical process to which we owe almost all the inductive conclusions we draw in daily life. When a man is shot through the heart, it is by this method that we know that is was the gun-shot which killed him: for he was in the fullness of life immediately before, all circumstances being the same, except the wound.

Though they are different, Mill says that the Methods of Agreement and of Difference have in common that they are both methods of attempted elimination.

> The Method of Agreement stands on the ground that whatever can be eliminated, is not connected with the phenomenon by any law. The Method of Difference has for its foundation, that whatever cannot be eliminated, is connected to the phenomenon by a law.

These two Methods can be combined in various ways. Suppose, for example, that we look at all the cases in which a occurs, and find that A is always present as an antecedent. The Method of Agreement tells us that A is sufficient for a. Suppose we want to know further whether A is necessary. Ideally, we would take one of our cases, remove the A, and see whether this removes the a. If we can do this, and if a is in fact removed by the removal of A, then by the Method of Difference we can know that A is not only sufficient but necessary for a.

However it often happens that we are not able to perform that crucial experiment, in which case, says Mill, we must resort to an indirect approach. Using some rather confusing terminology he calls this the **Indirect Method of Difference** or, equivalently, the **Joint Method of Agreement and Difference**, and then adds that it is not really the Method of Difference at all, but rather a double use of the Method of Agreement. What he has in mind is this. We first, with a use of the Method of Agreement, discover that A is sufficient for a. Then, looking at negative cases, we use the Method of Agreement again to discover that non-A is sufficient for *non-a*. Putting these together we get the same result as we would have gotten from the Method of Difference, had we been able to use it. For from our positive cases we will have, for example, A B C $a\ b\ c$. And our study of the negative cases tells us that if we had been able to remove A from A B C we would have had B C $b\ c$, which is what is required for the Method of Difference.

THE CASE OF DEW

As an example of the Joint Method, Mill givesan account of the discovery of the cause of dew, which was made, Mill says, by 'the late Dr. Wells.' (This example uses the Method of Concomitant Variations as a substitute for the Method of Agreement on the negative cases, and is discussed below.) In investigating the cause of dew we must first, of course, be clear about just what dew is, distinguishing it from rain, fog, etc. (Remember that England has a damp climate.) This leads us to connect dew with analogous phenomena such as "the moisture which bedews a cold metal or stone when we breathe upon it; that which appears on a glass of water fresh from the well in hot weather; that which appears on the inside of windows when sudden rain or hail chills the external air; . . . all these instances agree in one point, the coldness of the object dewed in comparison with the air in contact with it." This allows us to use the **Method of Agreement**. (We can use a thermometer to check the coldness theory in the case of nocturnal dew.)

But this, says Mill, just shows a connection between coldness and dew; it doesn't say which is cause and which effect. We must look for negative cases, so as to bring in the **Method of Difference**. One negative case occurs when we discover that dew forms on a sheet of glass under circumstances in which it will not form on a sheet of polished metal. We cannot yet invoke the **Method of Difference** because there are lots of respects in which glass and metal differ, and this Method requires us to narrow things down to just a single difference. But if we could find a quality shared by glass and all the things on which dew does fall, and not by metal and other things upon which it does not, then the **Method of Difference** could be used.

Here recourse is had to the **Method of Concomitant Variations**; to start, we look at different substances and find that the less the substance conducts heat, the greater the quantity of dew that falls on it.

But we find further that the surface quality (rough or smooth) of the object makes a difference. We bring in **Concomitant Variations** again, learning that the surfaces that best radiate heat from the surface are the ones that collect the most dew.

Putting these two discoveries together, we see that the crucial property is that of losing heat from the surface faster than it can be resupplied from within. Dew falls on objects that have this property, and not on those that don't. And this sets us up for the **Indirect Method of Difference**. We take one of the objects on which dew falls and now say that if we were to remove the heat losing property from that object then dew wouldn't fall on it.

We can now return to the question whether dew is the cause or effect of dew. The heat losing property, which we now know to be responsible for dew, also accounts for the coldness, so we must say that the dew depends on the coldness rather than the other way around.

And this conclusion, says Mill, can be confirmed by the **Method of Difference**. It is observed, he says, that dew does not come on a cloudy night, but does if the clouds withdraw. Open communication with the sky thus, by the **Method of Difference**, causes dew. And this fits in since it is a known property of clouds, as of other objects, that they radiate heat; they thus keep up the temperature of the things on which dew might otherwise fall. (This summary should surely earn the writer at least a B+ on Questions 2 and 3 of the 1873 University of Toronto Exam, even if not in a contemporary science course.)

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The point of the **Method of Residues** is to make use of what has been learned from previous inductions. Suppose we already know that B causes *b* and that C causes *c*, and that we now discover that A B C gives *a b c*. We 'subduct' the known effects of B and C, and conclude that A causes *a*. One of Mill's examples of this Method is of no little importance even today:

> To add one more example: those who assert, what no one has shown any real ground for believing, that there is on one human individual, one sex, or one race of mankind over another, an inherent and inexplicable superiority in mental faculties, could only substantiate their proposition by subtracting from the differences of intellect which we in fact see, all that can be traced by known laws either to the ascertained differences of physical organization, or to the differences which have existed in the outward circumstances in which the subjects of comparison have hitherto been placed. What these causes might fail to account for, would constitute a residual phenomenon, which and which alone would be evidence of an ulterior original distinction, and the measure of its amount. But the assertors of such supposed differences have not provided themselves with these necessary logical conditions of the establishment of their doctrine.

The last Method, **Concomitant Variations**, is needed, says Mill, when for one reason or another, it is impossible totally to remove some the antecedents, for example, the earth and the moon when we are studying the tides. But in such cases, though we cannot totally remove them, we can at least vary them and see if other things vary accordingly. We can, for example, observe how varying positions of the moon produce varying positions of the ocean with respect to the shore, and thus conclude that the moon causes the tides.

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More recent writers have considered Mill's Methods to be more limited in scope than did Mill himself, and current state of the art in experimental method and design, in statistics, and in the philosophy of science is at a level of technical sophistication quite beyond Mill's range. Even so, the Methods still have their place. In their famous logic textbook (*An Introduction to Logic and Scientific Method*, New York, 1934) the philosophers Morris R. Cohen and Ernest Nagel reach a balanced view. For example, discussing the Method of Agreement they say:

- Cohen and Nagel (1834)

> The method of agreement is therefore useless as a method of discovery and fallacious as a canon of proof. Has it then no value? It has a limited value if stated negatively: *Nothing can be the cause of a phenomenon which is not a common circumstance in all instances of the phenomenon.* (p. 255)

They make similar comments about the other methods, which all assume, what is not ordinarily the case, that a problem comes to us already broken down into 'circumstances' A B C, etc., that we already have a list of possible hypotheses, and that this list includes the true one. But the Methods do not tell us how to define the circumstances nor how to dream up the hypotheses, and we can never know for sure that the true one is on our list. But even so, the negative value the Methods have is still important.

> But while the methods we have discussed have the defects pointed out, they are of undoubted value in the process of attaining truth. For in eliminating false hypotheses, they narrow the field within which true ones may be found. And even where these methods may fail to eliminate all irrelevant circumstances, they enable us with some degree of approximation to so establish the conditions for the occurrence of a phenomenon, that we can say one hypothesis is logically preferable to its rivals. (p. 267)

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Close Reasoners finding themselves under an obligation to write an essay may appreciate the advice contained in Mill's account in the *Autobiography* of his writing of the *Logic*.

- John Stuart Mill (*Autobiography*, 1873)

> The first use I made of the leisure which I gained by disconnecting myself from the Review [Mill had been editor of the *Westminster Review*] was to finish the Logic. In July and August 1838, I had found an interval in which to execute what was still undone of the original draft of the Third Book. . . . the remainder of the work [was drafted] in the summer and autumn of 1840. From April following, to the end of 1841, my spare time was devoted to a complete re-writing of the book from its commencement. It is in this way that all my books have been composed. They were always written at least twice over; a first draft of the entire work was completed to the very end of the subject, then the whole begun again *de novo;* but incorporating in the second writing, all sentences and parts of sentences of the old draft, which appeared as suitable to my purpose as anything which I could write in lieu of them. I have found great advantages in this system of double redaction. It combines, better than any other mode of composition, the freshness and vigour of the first conception, with the superior precision and completeness resulting from prolonged thought. In my own case, moreover, I have found that the patience necessary for a careful elaboration of the details of composition and expression, costs much less effort after the entire subject has been once gone through, and the substance of all that I find to say has in some manner, however imperfect, been got upon paper. The only thing which I am careful, in the first draft, to make as perfect as I am able, is the arrangement. If that is bad, the whole thread on which the ideas string themselves becomes twisted; thoughts placed in a wrong connexion are not expounded in a manner that suits the right, and a first draft with this original vice is next to useless as a foundation for the final treatment. (Chapter VII)

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